

QUIZ 15 SOLUTIONS: LESSON 21
OCTOBER 20, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Estimate the change in z at $(2, -1)$ if $\frac{\partial z}{\partial x} = 2xy^2$, $\frac{\partial z}{\partial y} = 2x^2y$ given $\Delta x = -.01$ and $\Delta y = .01$.

Solution: Δz is the change in z and it is estimated by

$$\begin{aligned}\Delta z &= \frac{\partial z}{\partial x}(2, -1)\Delta x + \frac{\partial z}{\partial y}(2, -1)\Delta y \\ &= [2(2)(-1)^2](-.01) + [2(2)^2(-1)](.01) \\ &= 4(-.01) - 8(.01) \\ &= -12(-.01) \\ &= \boxed{-.12}\end{aligned}$$

2. [6 pts] A company produces boxes with square bases. Suppose they initially create a box that is 12 in tall and 6 in wide but they want to decrease the box's width by 1 in. Estimate how they must change the height so the box stays the same volume.

Solution: Let w be the width and h be the height of the box. Because this box has a square base, its volume is given by $V = w^2h$. We are given

$$w = 6, \quad h = 12, \quad \Delta w = -1.$$

Further, because we don't want to volume of the box to change, we must have $\Delta V = 0$. The goal is to find Δh .

We know that

$$V_w = 2wh \text{ and } V_h = w^2.$$

Then, evaluating at the point $(w, h) = (6, 12)$, we have

$$V_w(6, 12) = 2(6)(12) = 144$$

and

$$V_h(6, 12) = 6^2 = 36.$$

By our approximation formula, we have

$$\Delta V = V_w(6, 12)\Delta w + V_h(6, 12)\Delta h.$$

Substituting what we know, this becomes

$$0 = (144)(-1) + (36)\Delta h.$$

We want to solve for Δh , so we get

$$144 = 36\Delta h \Rightarrow \boxed{\Delta h = 4}.$$

Thus, the company will need to increase the height of the box by 4 inches.